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SELECTION OF THE OPTIMAL DISTRIBUTION OF HEAT-TRANSFER AGENT FLOWS IN A VENTILATED CASSETTE RADIOELECTRONIC APPARATUS

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UDC 536.24

An algorithm for determining the optical distribution of heat-transfer agent flows is proposed, and recommendations regarding the solution of the optimization problem are given.

As the analysis of different structures shows, combined cooling systems are increasingly more often used in radioelectronic apparatus (REA). The most efficient, economical, and simple cooling systems are those based on conductive outflow through heat sinks. However, the higher density of the assembly and the higher power of the electronic equipment make it necessary to use specific methods for removing heat. In many cases the amount of heat removed is increased by natural or forced ventilation [1, 2].

In this work we study electronic apparatus with a cassette construction with forced ventilation. The REA block is assembled from identical boards with modules arranged on them or integrated circuits. The metallic frames and heat sinks play the role of structural elements and raise the effective thermal conductivity of the heated zone. In the general case, the heat sources in such REA are distributed nonuniformly, and each source occupies an arbitrary region and has an arbitrary capacity. In REA of the cassettetype, however, the problem of modeling the heat sources is simplified by the fact that the cassettes or groups of cassettes occupy volumes in the form of steps with a rectangular cross section, the power in which may be assumed to be uniformly distributed.

Experience in constructing REA shows that the volume of stored air or other heat-transfer agent is limited, and this does not permit efficient cooling of the entire volume of the apparatus [3]. In the case of a stepped distribution of heat sources, some zones do not require special cooling, so that it is desirable to ventilate the heated zone only in the regions with the highest thermal loads.

Channels in REA of a cassette form have a rectangular profile, so that the regions of convection, just as the sources of heat, will have the form of rectangular steps.

There arises the problem of selecting the thermal and mathematical model of the apparatus described for the analysis of the temperature field as well as for the formulation and solution of the question of optimal distribution of heat-transfer agent flows between the channels of the apparatus in order to achieve the best, with respect to a definite criterion thermal state.

Thermal Model. The thermal model of the object studied is a particular case of the generalized model studied in [4]. The heated zone of the REA is represented in the form of a uniform anisotropic parallelepiped with effective coefficients of thermal conductivity λ_x, λ_y, λ_z along the corresponding axes (Fig. 1). The heat flux from the surfaces of the parallelepiped escapes into the surrounding medium, and is also carried away by convective air flows, blown through the apparatus. Thus the thermal model can be viewed as a system of two bodies: a uniform anisotropic parallelepiped with a step distribution of heat sources and regions of convection and a moving medium in the channels of the heated zone.

Leningrad Institute of Precision Mechanics and Optics. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 50, No. 2, pp. 249-255, February, 1986. Original article submitted January 14, 1985.

 0022-0841/86/5002-0182$12.50

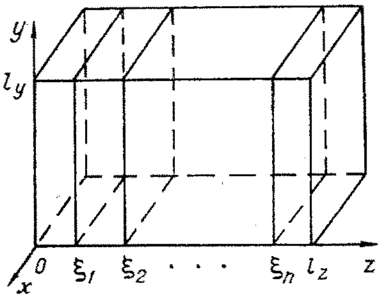

Fig. 1. Thermal model of an apparatus of the cassette type.

<u>Mathematical Model</u>. The analysis of the temperature field of the model reduces to the solution of a system of two equations: the heat-conduction equation for a parallelepiped including convection and the heat-transfer equation of air in a channel:

$$\lambda_x \frac{\partial^2 \vartheta}{\partial x^2} + \lambda_y \frac{\partial^2 \vartheta}{\partial y^2} + \lambda_z \frac{\partial^2 \vartheta}{\partial z^2} = -\sum_{i=1}^{n} 1\{\mathcal{U}_i(z)\}\,[W_i - \alpha_{Vi}(\vartheta - \vartheta_f)]; \tag{1}$$

$$\sum_{i=1}^{n} \alpha_{Vi}(\vartheta - \vartheta_f)\, 1\{\mathcal{U}_i(z)\} = \sum_{i=1}^{n} \omega_i \frac{\partial \vartheta_f}{\partial y}\, 1\{\mathcal{U}_i(z)\}; \tag{2}$$

$$\left[\lambda_k \frac{\partial \vartheta}{\partial k} \pm \alpha\Big|_{0k}^{1k} \vartheta\right]_{k=0}^{k=l_k} = 0; \quad k = x,\ y,\ z; \tag{3}$$

$$\vartheta_f|_{y=0} = \vartheta_{\text{in}}. \tag{4}$$

Here $\vartheta(x, y, z)$ is the overheating of the heated zone relative to the medium, K; $\vartheta_f(x, y, z)$ overheating of the air moving in the channel, K; $1\{\mathcal{U}_i(z)\}$ symbolic notation for the unit function defined as follows:

$$1\{\mathcal{U}_i(z)\} = \begin{cases} 1 & \text{for the i-th zone,} \\ 0 & \text{outside the i-th zone} \end{cases} \tag{5}$$

$$\alpha_V = 2\alpha_\ell/(\delta + b); \tag{6}$$

α_V is the coefficient of volume convection, $W/m^3 \cdot K$; δ, cassette thickness, m; b, channel width, m; ℓ_x, ℓ_y, ℓ_z, parallelepiped dimensions along the corresponding axes, m; W_i, volume density of heat sources in the i-th zone, W/m^3; α_ℓ, heat-transfer coefficient in the channel, $W/m^2 \cdot K$; α_{0k}, α_{1k}, heat-transfer coefficients from the corresponding faces, $W/m^2 \cdot K$;

$$\omega_i = \frac{C_p G_{\rho i}}{l_x(\xi_i - \xi_{i-1})}, \quad i = 1,\ 2,\ \ldots,\ n, \tag{7}$$

where ξ_i is the coordinate of the end of the i-th zone along the z axis ($\xi_0 = 0$, $\xi_n = \ell_z$); and C_p is the specific heat capacity of the heat-transfer agent moving inside the parallelepiped, J/kg·K; and $G_{\rho i}$ is the flow rate of the heat-transfer agent in the i-th zone, kg/sec.

Thus, the boundary-value problem (1)-(4) consists of a system of two linear inhomogeneous partial differential equations with homogeneous boundary conditions. It is solved by the method of successive averaging [5]. We present without derivation the final working formulas:

$$\bar{\vartheta}_i(x,\ y,\ z) = B_{2i-1} e^{p_{2i-1} y} + B_{2i} e^{p_{2i} y} +$$

$$+ (\lambda_x p_{xi}^2 \bar{\vartheta}_{yi} - W_{\text{in}.i}) \frac{y}{\omega_i} + \left[\lambda_x p_{xi}^2 \left(\frac{\lambda_y}{\omega_i^2} + \frac{1}{\alpha_{Vi}}\right) - \frac{\alpha_{0y}\psi_{0y}}{\omega_i}\right] \bar{\vartheta}_{yi} - \left(\frac{\lambda_y}{\omega_i^2} + \frac{1}{\alpha_{Vi}}\right) W_{\text{in}.i} + \vartheta_{\text{in}}. \tag{8}$$

Here $\bar{\vartheta}_i(x, y, z)$ is the approximate value of the overheating $\bar{\vartheta}_i(x, y, z)$ of the heated zone at the point with the coordinates (x, y, z) belonging to the i-th zone;

$$\bar{\vartheta}_{fi}(x, y, z) = \vartheta_{in} + \frac{\lambda_y \frac{\partial \bar{\vartheta}_i}{\partial y} - W_{in \cdot i}\, y + \bar{\vartheta}_{yi}(\lambda_x p_{xi}^2\, y - \alpha_{0y}\psi_{0y})}{\omega_i}, \tag{9}$$

$\bar{\vartheta}_{fi}(x, y, z)$ is the approximate value of the overheating of the heat-transfer agent, moving in the channel, at a point with the coordinates (x, y, z).

The following notation has been adopted in the expressions (8) and (9):

$$p_{2i(2i-1)} = -\frac{\alpha_{Vi}}{2\omega_i}\left(1 \mp \sqrt{1 + \frac{4\omega_i^2}{\alpha_{Vi}\lambda_y}}\right);$$

$$B_{2i-1} = \frac{(Y_i F_i + N_i Q_i)\,\bar{\vartheta}_{yi} + K_i Q_i - Y_i E_i}{U_i Q_i - D_i Y_i};$$

$$B_{2i} = \frac{E_i - F_i \bar{\vartheta}_{yi} - D_i B_{2i-1}}{Q_i};$$

$$D_i = (\lambda_y p_{2i-1} + \alpha_{1y})\, e^{p_{2i-1} l_y}; \quad Q_i = (\lambda_y p_{2i} + \alpha_{1y})\, e^{p_{2i} l_y};$$

$$F_i = \frac{\lambda_x p_{xi}^2}{\omega_i}\left[\lambda_y + \alpha_{1y} l_y + \frac{\alpha_{1y}\lambda_y}{\omega_i} + \frac{\alpha_{1y}\omega_i}{\alpha_{Vi}} - \frac{\alpha_{0y}\alpha_{1y}\psi_{0y}}{\lambda_x p_{xi}^2}\right];$$

$$U_i = \lambda_y p_{2i-1} - \alpha_{0y}; \quad Y_i = \lambda_y p_{2i} - \alpha_{0y};$$

$$E_i = \frac{W_{in\,i}}{\omega_i}\left(\lambda_y + \alpha_{1y} l_y + \frac{\alpha_{1y}\lambda_y}{\omega_i} + \frac{\alpha_{1y}\omega_i}{\alpha_{Vi}}\right) - \alpha_{1y}\vartheta_{in}$$

$$N_i = \frac{\lambda_x p_{xi}^2}{\omega_i}\left(-\lambda_y + \frac{\alpha_{0y}\lambda_y}{\omega_i} + \frac{\alpha_{0y}\omega_i}{\alpha_{Vi}} - \frac{\alpha_{0y}^2\psi_{0y}}{\lambda_x p_{in}^2}\right);$$

$$K_i = \frac{W_{in\,i}}{\omega_i}\left(\lambda_y - \frac{\alpha_{0y}\lambda_y}{\omega_i} - \frac{\alpha_{0y}\omega_i}{\alpha_{Vi}}\right) + \alpha_{0y}\vartheta_{in};$$

$$W_{in\,i} = \frac{\omega_i \alpha_{Vi}\vartheta_{in}\psi_{fy}}{\alpha_{Vi} l_y + \omega_i \psi_{fy}};$$

$$p_{xi}^2 = b_x^2 + \frac{\alpha_{Vi}}{\lambda_x}\left(1 - \frac{1}{1 + \frac{\omega_i\psi_{fy}}{\alpha_{Vi} l_y}}\right);$$

$$b_x^2 = \frac{\alpha_{1y}\psi_{1y} + \alpha_{0y}\psi_{0y}}{\lambda_x l_y};$$

$$\bar{\vartheta}_{yi} = \bar{\vartheta}_{ixy}\frac{\lambda_z p_{zi}^2}{\lambda_x p_{xi}^2}\left(1 + A_{2i-1} e^{p_{xi}x} + A_{2i} e^{-p_{xi}x}\right);$$

$$p_{zi} = b_z^2 + \frac{\alpha_{Vi}}{\lambda_z}\left(1 - \frac{1}{1 + \frac{\omega_i\psi_{fy}}{\alpha_{Vi} l_y}}\right);$$

$$b_z^2 = \frac{\alpha_{1x}\psi_{1x} l_y + \alpha_{0x}\psi_{0x} l_y + \alpha_{0y}\psi_{1y} l_x + \alpha_{0y}\psi_{0y} l_x}{\lambda_z l_x l_y};$$

$$A_{2i} = \frac{\alpha_{1x} T_2 + \alpha_{0x} E_x T_3}{T_2 T_4/E_x - E_x T_1 T_3}; \quad E_x = e^{p_{xi} l_x};$$

$$A_{2i-1} = \frac{\alpha_{0x} T_4/E_x + \alpha_{1x} T_1}{T_2 T_4/E_x - E_x T_1 T_3};$$

$$T_1 = \lambda_x p_{xi} + \alpha_{0x}; \quad T_2 = \lambda_x p_{xi} - \alpha_{0x}; \quad T_3 = \lambda_x p_{xi} + \alpha_{1x};$$

$$T_4 = \lambda_x p_{xi} - \alpha_{1x};$$

$$\bar{\vartheta}_{ixy} = C_{2i-1} e^{p_{zi}(z - s_i)} + C_{2i} e^{-p_{zi}(z - s_i)} + \frac{W_i + W_{in\,i}}{\lambda_z p_{zi}^2};$$

$$s_i = \frac{\xi_i + \xi_{i-1}}{2}; \quad i = 1, 2, \ldots, n.$$

The values of the coefficients of nonuniformity of the temperature field in the first approximation can be evaluated from the formulas

$$\psi_{mk} = \frac{1}{1 + \mathrm{Bi}_{mk}/3}; \quad \mathrm{Bi}_{mk} = \frac{\alpha_{mk} l_k}{2\lambda_k}; \quad m = 0, 1;$$
$$k = x, y, z; \quad \psi_{fy} = 2.$$

The coefficients C_{2i-1} and C_{2i} are determined from the solution of the system of equations arising when the solution is substituted into the boundary conditions.

Formulation and Solution of the Optimization Problem. The overall flow rate of air, which is usually fixed, can be distributed differently between the cassettes of the REA. We shall call the distribution that gives the best thermal state with respect to one or another point criterion optimal. The question of the selection of the criterion for evaluating the thermal state is very complicated and cannot be solved uniquely for all possible cases. Most often the optimal state is the thermal state in which the maximum operational reliability of the apparatus is achieved. In this case, if it is assumed that all elements of the aparatus are equally reliable, we arrive at the requirement of minimization of the total temperature of all elements. Other criteria can also be used. In addition to lowering the temperature, one also strives to equalize the temperatur field in order to reduce the temperature gradients. A good criterion is the maximum value of the temperatures of all elements, since the minimization of the maximum temperature indirectly solves the questions of both equalization of the temperature field and minimization of the temperatures.

The problem of the optimal distribution of the heat-transfer agent is studied, in particular, in [3]. In [3] particular constructions of REA are studied and recommendations, which are of practical value for the designers of the apparatus, are given. However, the problems encountered in practice cannot always be reduced to cases in which simple recommendations can be given (for example, the distribution of the heat-transfer agent proportionally to the square root of the dissipated power [3], etc.). In addition, once a set of simple recommendations is in hand, it is convenient to make a "manual" design, while in order to create a computer-assisted design systems (CADS) it is necessary to have rigorously formalized procedures for selecting the optimal parameters of the structure.

The problem addressed in this work encompasses the discussion of different formulations of the problem of the optimal distribution of heat-transfer agent flows and methods for solving it for use in CADS.

We denote by $\bar{t}_{xy}(z)$ the average temperature along the x and y axes of the cassette apparatus and by $\bar{t}$ the temperature averaged over the volume. Then the foregoing criteria for evaluating the thermal state can be written as

$$f_1(G_1, G_2, \ldots, G_n) = \bar{t}, \tag{10}$$

$$f_2(G_1, G_2, \ldots, G_n) = \sum_{i=1}^{m} [t_{xy}(z_i) - \bar{t}]^2, \tag{11}$$

$$f_3(G_1, G_2, \ldots, G_n) = \max_z t_{xy}(z), \tag{12}$$

where G_j is the flow rate of the heat-transfer agent through the j-th zone of the apparatus; m is the number of points on the z axis at which the temperature is calculated; and n is the number of zones of the cassette apparatus.

The optimization problem is stated in the following manner. It is necessary to determine the flow rate of the heat-transfer agent in each zone. In so doing, the criterional function written in the form (10), (11), or (12) must be minimized. The problem is solved under the following restrictions:

$$\sum_{j=1}^{n} G_j = G_\Sigma, \tag{13}$$

$$G_j \geqslant 0, \quad j = 1, 2, \ldots, n, \tag{14}$$

where G_Σ is the fixed total flow rate of the heat-transfer agent.

The optimization problem formulated in this manner is a general problem in nonlinear programming [6]. The choice of one or another criterional function is determined by the specific structure of the apparatus, as well as by the solution method. To realize the problem studied, one of the methods used in solving the general problem of nonlinear programming — the "moving tolerance" method [6] — was selected. Optimization was carried out with different sets of starting data and all three forms of the criterional function. We shall point out some of the general considerations without presenting the specific results.

1. The solution method used enables writing down the restrictions on the parameters (13) and (14) in an explicit form, which facilitates the application of the method.

2. The restrictions (14) follow from the physical meaning of the problem. They must be given; otherwise absurd results with a negative flow rate can be obtained.

3. Since the optimization method is such that in the course of the solution calculations of the criterional function with inadmissible values of the parameters could be performed (for example, negative flow rates of the heat-transfer agent), the order of the calculation for the inadmissible region of variation of the parameters must be predetermined. In the problem udner study, for $G_j < 0$ the calculation of the temperature field was carried out with $G_j = 0$.

4. The presence of restrictions (13) substantially complicates the search for admissible values of the parameters, so that their presence in the formulation of the problem is undesirable. For the problem at hand the restrictions (13) can be written in the form

$$\sum_{j=1}^{n} G_j \leqslant G_\Sigma, \tag{15}$$

which, though formally destroys the formulation of the problem, accelerates the solution and yields approximately the same result.

Another variant is the exclusion of one of the parameters, for example,

$$G_n = G_\Sigma - \sum_{j=1}^{n-1} G_j. \tag{16}$$

Then the number of parameters sought decreases by one, and the restrictions are eliminated.

5. The optimization problem can be formulated in a different manner: determine the minimum flow rate of the heat-transfer agent with restrictions on the temperatures, i.e.,

$$\sum_{j=1}^{n} G_j \to \min \quad \text{for} \quad t_j \leqslant t_{\mathrm{adm}}. \tag{17}$$

In this formulation, the problem appears to be even more interesting, but its realization requires more work.

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ENERGY LOSSES IN COOLING OF CRYOGENIC CURRENT LEADS

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UDC 537.312.62:621.315.626.3

The effect of cooling system construction upon energy expended in cooling cryogenic current leads is studied for self-regulating and forced cooling by liquid helium.

When cryogenic equipment is utilized a cooling agent at temperature T_r is supplied continuously from a cryogenic device (refrigerator or liquifier) to the cryostat. Within the equipment being cooled it circulates at a higher temperature T_c, due to heat exchange with elements of the equipment, one end of which is at a low temperature T_0, while the other end is at a higher temperature T_ℓ. Usually the conditions of use require that the value of T_ℓ coincide with the temperature of the surrounding medium T_m. In a closed cycle system the cooling agent temperature decreases from a temperature T_c to T_r, which requires use of energy. The amount of energy thus used determines the cost of the equipment operation.

The goal of the present study is to analyze the cooling system with regard to energy expenditure in the cryogenic device for self-regulating and forced cooling of current leads. Some aspects of this problem for self-regulating cooling have already been considered in [1-4].

The value of the energy expenditure is determined by the product of the difference of the cooling agent exergies ΔE at temperatures T_c and T_r times the coolant mass flow rate G. The exergy difference is given by the expression

$$\Delta E(T_c, T_r) = [T_m S(T_c) - h(T_c)] - [T_m S(T_r) - h(T_r)].$$

With self-regulating cooling $T_r = T_0 = T_S$. Using known thermodynamic relationships for an ideal gas:

$$S(T_S) - S_L = \frac{r}{T_S}, \quad (1)$$

$$h(T_S) - h_L = r, \quad (2)$$

$$h = C_r T, \quad (3)$$

$$S(T_c) - S(T_0) = C_p \ln \frac{T_c}{T_0}, \quad (4)$$

one can calculate ΔE. Comparison with the data of [6] shows that the error in the calculation for helium does not exceed 5%.

We will study the behavior of the function $\omega = w/I(T_c)$ for self-regulated cooling of current leads, wherein a segment with temperature in the interval (T_0, T_c) is cooled by a flow rate G, while the remainder with temperature in the interval (T_c, T_ℓ) is cooled by a

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 50, No. 2, pp. 255-260, February, 1986. Original article submitted November 27, 1984.

0022-0841/86/5002-0187$12.50